# 2 <br> <br> Mathematics Applications 

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11 Two forces of magnitudes 8 and 16 kg .wt. and the measure of their included augle is $120^{\circ}$ If these two forces act at a body, then the direction of motion of the body makes an angle of measure $\qquad$ with the smaller force.
(a) $30^{\circ}$
(b) $90^{\circ}$
(C) $60^{\circ}$
(d) $45^{\circ}$

2 Two forces of equal magnitude and intersecting at a point. The measure of the angle between the two forces is $120^{\circ}$ and the magnitude of each is 6 N ., then the magnitude of their resultant $=$ $\qquad$
(a) 12
(b) $6 \sqrt{3}$
(c) 6
(d) $12 \sqrt{3}$

3 FN . and K N . are the magnitudes of two forces where $\mathrm{F}>\mathrm{K}$ If the smallest and the greatest value of their resultant are 5,9 newton respectively, then $5 \mathrm{~F}-2 \mathrm{~K}=$ N .
(a) 53
(b) 31
(C) 49
(d) 4

4 A body of weight 20 N . is placed on a smooth inclined plane makes an angle of measure $30^{\circ}$ with the horizontal, then the component of the weight in direction perpendicular to the plane $=$ $\qquad$
(a) 10
(b) 20
(C) $10 \sqrt{2}$
(d) $10 \sqrt{3}$

5 Forces of magnitudes $8,4 \sqrt{3}, 6 \sqrt{3}, 14$ newton act at a point. The measure of the angle between the first and second force is $30^{\circ}$ and between the second and third is $120^{\circ}$ and between the third and fourth is $90^{\circ}$ in one cyclic order, then the magnitude of their resultant $=$ $\qquad$
(a) 4
(b) 6
(C) 8
(d) 7

6 Two forces of magnitudes $3, F$ newton and measure of the angle between them is $\frac{2 \pi}{3}$ if their resultant is perpendicular to the first force, then $\mathrm{F}=$ newton.
(a) 1.5
(b) 3
(C) $\sqrt[3]{2}$
(d) 6

## 2 Answer the following questions:

1 A force of magnitude 18 newton acts in south direction. Find its two components in directions of $60^{\circ}$ East of South and $30^{\circ}$ West of South.
(2 marks)
2 Three coplanar forces of magnitudes $1,2, \sqrt{3}$ newton act at M , their directions are $\overrightarrow{\mathrm{MA}}$, $\overrightarrow{\mathrm{MB}}$ and $\overrightarrow{\mathrm{MC}}$ respectively where $\mathrm{m}(\angle \mathrm{AMB})=60^{\circ}, \mathrm{m}(\angle \mathrm{BMC})=30^{\circ}, \mathrm{m}(\angle \mathrm{AMC})=90^{\circ}$ Find the resultant.
(2 marks)

## Test <br> 2

1 Choose the correct answer from those given :
1 The resultant of two forces 6,8 newton is 10 N ., then the measure of the angle between their directions $=$ $\qquad$ -
(a) 60
(b) 90
(C) 120
(d) 150

2 Two forces intersecting at a point, their magnitudes 7 and F newton and their resultant bisects the angle between them, then $(\mathrm{F}-1)=$ $\qquad$ N .
(a) 8
(b) 7
(c) 6
(d) 5

3 In the opposite figure :
The force $\vec{R}$ is resolved into two components $\vec{F}_{1}$ and $\vec{F}_{2}$ , then $\mathrm{F}_{1}=$ $\qquad$ newton.
(a) $12 \cos 75^{\circ}$
(b) $12 \cos 45^{\circ}$
(C) $6 \csc 45^{\circ}$
(d) $6 \csc 75^{\circ}$


## 4 In the opposite figure :

If the resultant of the shown forces acts in direction of $y$-axis, then $F=$ $\qquad$ N .
(a) 2
(b) 6
(c) 8
(d) 14


5 The magnitudes of two forces are 5 and 10 newton and their resultant is perpendicular on the smaller force. If the measure of angle between the two forces is $\alpha$ and their resultant is $\mathbb{R}$, then
(a) $\alpha=60^{\circ}, \mathbb{R}=10 \sqrt{3} \mathrm{~N}$.
(b) $\alpha=120^{\circ}, \mathbb{R}=10 \sqrt{3} \mathrm{~N}$.
(c) $\alpha=60^{\circ}, \mathbb{R}=5 \sqrt{3} \mathrm{~N}$.
(d) $\alpha=120^{\circ}, \mathbb{R}=5 \sqrt{3} \mathrm{~N}$.

## 6 In the opposite figure :

A body of wiehgt 260 gm.wt. and $\tan \theta=\frac{5}{12}$,
$\mathrm{W}_{1}, \mathrm{~W}_{2}$ are magnitudes of the two components in direction of the inclined plane downward and perpendicular to the plane, then

(a) $\mathrm{W}_{1}=120$ gm.wt., $\mathrm{W}_{2}=50 \mathrm{gm} . \mathrm{wt}$.
(b) $\mathrm{W}_{1}=260 \mathrm{gm} . \mathrm{wt} ., \mathrm{W}_{2}=65 \mathrm{gm} . \mathrm{wt}$
(c) $\mathrm{W}_{1}-\mathrm{W}_{2}=70$ gm.wt.
(d) $\mathrm{W}_{1}+\mathrm{W}_{2}=340 \mathrm{gm} . \mathrm{wt}$.

## 2 Answer the following questions:

## 1 In the opposite figure :

If the force of magnitude 40 N . is resolved into two components $\vec{F}_{1}$ and $\overrightarrow{\mathrm{F}}_{2}$ as shown in the figure.


Find the two component magnitudes $\mathrm{F}_{1}, \mathrm{~F}_{2}$
(2) The magnitudes of three forces are $10,20,30$ newton acting at one point. The first acts due east, the second makes an angle of measure $30^{\circ}$ west of the north and the third makes an angle of measure $60^{\circ}$ south of the west. Find the magnitude and the direction of their resultant.

## Answers of Mathematics Applications

## Answers of Test 1

1
3 (b)
(d)
5 (a)
6 (d)

2 2 The two components are perpendicular
$\therefore \mathrm{F}_{1}=18 \cos 60^{\circ}$
$=9$ newton

$$
\begin{aligned}
, \mathrm{F}_{2} & =18 \sin 60^{\circ} \\
& =9 \sqrt{3} \text { newton }
\end{aligned}
$$



South

2 Consider $\overrightarrow{\mathrm{OX}}$ is the direction of the first force.

$$
\begin{aligned}
& \mathrm{X}=1 \times \cos 0^{\circ}+2 \cos 60^{\circ}+\sqrt{3} \cos 90^{\circ} \\
&=1 \times 1+2 \times \frac{1}{2}+\sqrt{3} \times 0=2 \\
& Y=1 \times \sin 0^{\circ}+2 \times \sin 60^{\circ}+\sqrt{3} \sin 90^{\circ} \\
&=1 \times 0+2 \times \frac{\sqrt{3}}{2}+\sqrt{3} \times 1=2 \sqrt{3} \\
& \therefore \vec{R}=2 \vec{i}+2 \sqrt{3} \vec{j}, R=\sqrt{(2)^{2}+(2 \sqrt{3})^{2}}=4 \text { newton } \\
&, \tan \theta=\frac{2 \sqrt{3}}{2}=\sqrt{3} \\
&, \because X>0 \quad, Y>0
\end{aligned}
$$

$$
\therefore \theta=60^{\circ}
$$

$\therefore$ The magnitude of $\overrightarrow{\mathrm{R}}=4$ newton and its direction is $\overrightarrow{\mathrm{MB}}$

## Answers of Test 2

1 (b) 2 (c)
(d)
(4)
(d)
(6)

21 From the figure

$$
\begin{aligned}
& \sin \theta=0.8, \cos \theta=0.6 \\
& \therefore \frac{\mathrm{~F}_{1}}{\sin \left(90^{\circ}-\theta\right)}=\frac{\mathrm{F}_{2}}{\sin 90^{\circ}}=\frac{40}{\sin \left(180^{\circ}-\theta\right)} \\
& \therefore \frac{\mathrm{F}_{1}}{\cos \theta}=\frac{\mathrm{F}_{2}}{1}=\frac{40}{\sin \theta} \\
& \therefore \frac{\mathrm{~F}_{1}}{0.6}=\frac{\mathrm{F}_{2}}{1}=\frac{40}{0.8} \\
& \therefore \mathrm{~F}_{1}=30 \mathrm{~N} ., \mathrm{F}_{2}=50 \mathrm{~N} .
\end{aligned}
$$

(2) $\mathrm{X}=10 \cos 0^{\circ}+20 \cos 120^{\circ}+30 \cos 240^{\circ}=-15$

$$
Y=10 \sin 0^{\circ}+20 \sin 120^{\circ}+30 \sin 240^{\circ}=-5 \sqrt{3}
$$

$$
\therefore \stackrel{\rightharpoonup}{\mathrm{R}}=-15 \overrightarrow{\mathrm{i}}-5 \sqrt{3} \overrightarrow{\mathrm{j}}
$$

$$
\therefore \mathrm{R}=\sqrt{225+75}=10 \sqrt{3} \mathrm{~N} .
$$

$$
\tan \theta=\frac{y}{x}=\frac{-5 \sqrt{3}}{-15}=\frac{1}{\sqrt{3}}
$$

$$
, \because x<0, y<0
$$

$$
\therefore \theta=180^{\circ}+30^{\circ}=210^{\circ}
$$



Choose the correct answer from the given ones :
(1) The force is defined by
(a) its magnitude.
(b) its direction.
(c) the point of action.
(d) all the previous.
(2) Two forces act at a point. The magnitude of the two forces are 5, 3 newton and the angle between them $60^{\circ}$, then the magnitude of their resultant $=$ $\qquad$ newton.
(a) 2
(b) 5
(c) 7
(d) 8
(3) Two forces act at a point the magnitude of the two forces $8 \sqrt{3}, 8$ newton and the measure of the included angle between them $150^{\circ}$, then the magnitude of their resultant $=$ $\qquad$ newton.
(a) 64
(b) 32
(c) 16
(d) 8
( 4 ) Two perpendicular forces act at a point. The magnitude of the two forces 12,5 newton, then the magnitude of their resultant $=$ $\qquad$ newton.
(a) 17
(b) 7
(c) 13
(d) 14
( 5 ) Resultant of two forces 6 newton and 8 newton could be $\qquad$ newton.
(a) 20
(b) 15
(c) 12
(d) 1
( 6 ) The magnitude of two forces are $4,5 \mathrm{~N}$. They act at a point and cosine of their included angle is $\frac{-2}{5}$, then the magnitude of their resultant $R=$ $\qquad$ newtons.
(a) 15
(b) 5
(c) 20
(d) 25
(7) Two forces act at a point. The magnitude of the two forces are 6,3 newton and their resultant is perpendicular to one of them, then the magnitude of their resultant $=$ $\qquad$ newton.
(a) 3
(b) $3 \sqrt{3}$
(c) 6
(d) $6 \sqrt{3}$
( 8 ) Two forces enclosing between them an angle of measure $\theta$, then the magnitude of their resultant $\qquad$ ..
(a) increase as the value of $\theta$ increase.
(b) doubled as the value of $\theta$ doubled.
(c) increase as the value of $\theta$ decrease.
(d) don't change as change of the value of $\theta$
(9) In the opposite figure :

The magnitude of the resultant of the two forces in the figure $=$ $\qquad$ newton.
(a) 7
(b) 5
(c) 1
(d) $\sqrt{7}$

(10) In the opposite figure :

Magnitude of the resultant of the two forces $=$ $\qquad$ newton.
(a) 2 F
(b) F
(c) $\sqrt{3} \mathrm{~F}$
(d) zero

(11) The magnitude of the resultant of the two forces shown in the
 opposite figure is $\qquad$ of tho forces
(a) $\frac{1}{2} \mathrm{~F}$
(b) F
(c) $\sqrt{3} \mathrm{~F}$
(d) $\sqrt{5} \mathrm{~F}$
(12) If the resultant of the two forces $\mathrm{F}_{1}, \mathrm{~F}_{2}$ bisects the angle between them. Which of the following statements is true ?
(1) $\mathrm{F}_{1}=\mathrm{F}_{2}$
(2) $\overrightarrow{\mathrm{F}}_{1}=\overrightarrow{\mathrm{F}}_{2}$
(3) $\overrightarrow{\mathrm{R}}=\overrightarrow{\mathrm{F}_{1}}+\overrightarrow{\mathrm{F}_{2}}$
(a) only (1)
(b) only (1), (3)
(c) only (2), (3)
(d) All the previous.
(13) Two forces act at a point. The magnitude of the two forces are $\mathrm{F}, 2$ newton and the measure of the angle between them is $60^{\circ}$, if their resultant equal $2 \sqrt{3}$ newton , then $\mathrm{F}=$ $\qquad$ newton.
(a) 2
(b) 4
(c) 8
(d) 12
(14) The magnitude of two forces $\mathrm{F}, 2$ newton and the measure of their included angle $=\frac{2 \pi}{3}$ and the magnitude of their resultant is F newton, then $\mathrm{F}=\ldots \ldots \ldots .$. newton.
(a) 2
(b) 3
(c) 4
(d) $2 \sqrt{2}$
(15) Two forces of equal magnitudes, enclosing between them an angle of measure $\frac{\pi}{2}$ If the magnitude of their resultant is 8 N ., then the value of each force measured in newton is $\qquad$
(a) $2 \sqrt{2}$
(b) 4
(c) $4 \sqrt{2}$
(d) 8
(16) Two equal forces in magnitude, the magnitude of their resultant $=7 \sqrt{3}$ newton and the measure of the included angle is $\frac{\pi}{3}$, then the magnitude of each of them $=$ $\qquad$ newton.
(a) 3
(b) $5 \sqrt{3}$
(c) 5
(d) 7
(17) The magnitude of two forces F, F kg.wt., the magnitude of their resultant $24 \mathrm{~kg} . \mathrm{wt}$. and inclined to the first force by an angle of measure $30^{\circ}$ , then $\mathrm{F}=$ $\qquad$ kg.wt.
(a) 8
(b) $8 \sqrt{3}$
(c) $8 \sqrt{2}$
(d) 12
(18) Two forces of magnitudes 8 and $\mathrm{Fgm.wt}$. The measure of the angle between them is $\alpha \in] 0, \pi[$, their resultant bisects the included angle between them , then $\mathrm{F}=$ $\qquad$ gm.wt.
(a) 4
(b) 16
(c) $2 \sqrt{2}$
(d) 8
(19) Two forces of magnitudes $3, \mathrm{~F}$ newton and the measure of the angle between them is $120^{\circ}$. If their resultant is perpendicular to the first force, so the value of F in newton is $\qquad$
(a) 1.5
(b) 3
(c) $3 \sqrt{3}$
(d) 6
(20) The magnitude of two perpendicular forces are $(2 \mathrm{~F}-5)$ and $(\mathrm{F}+2)$ newton and the magnitude of their resultant if $3 \sqrt{5}$ newton, then $\mathrm{F}=$ newton.
(a) 7
(b) 4
(c) 6
(d) 3
(21) Two forces of magnitudes 6 N . and 10 N ., if the magnitude of their resultant is 14 N . , then the measure of the angle between the forces is $\qquad$
(a) $15^{\circ}$
(b) $30^{\circ}$
(c) $60^{\circ}$
(d) $45^{\circ}$
(22) Two equal forces, the magnitude of each of them is 6 N ., the magnitude of their resultant is 6 N ., then the angle between them equals $\qquad$
(a) $30^{\circ}$
(b) $60^{\circ}$
(c) $120^{\circ}$
(d) $150^{\circ}$
(23) Two forces of magnitudes 6 N . and 8 N ., if the magnitude of their resultant is 2 N . , then the measure of the angle between the two forces is $\qquad$ ..
(a) $30^{\circ}$
(b) $90^{\circ}$
(c) $180^{\circ}$
(d) $270^{\circ}$
(24) Magnitude of resultant of two forces of magnitudes $6,2.5$ newton is equal to 6.5 newton, then the angle between the two forces is $\qquad$
(a) an acute angle.
(b) an obtuse angle.
(c) a right angle.
(d) a straight angle.
(25) The magnitude of two forces are $2 \mathrm{~F}, 5 \mathrm{~F}$ newton and the measure of their included angle is $\theta$ and their resultant is 3 F , then $\theta=$ $\qquad$
(a) zero
(b) $60^{\circ}$
(c) $90^{\circ}$
(d) $180^{\circ}$
(26) Two forces of magnitudes 3 F and F newton and their resultant is 4 F newton , then the measure of the angle between them $=$ $\qquad$
(a) $60^{\circ}$
(b) $0^{\circ}$
(c) $180^{\circ}$
(d) $90^{\circ}$
(27) Two forces of magnitudes F and F act at a particle and their resultant is F , then the measure of the angle between the two forces $=$ $\qquad$
(a) $120^{\circ}$
(b) $60^{\circ}$
(c) $45^{\circ}$
(d) $90^{\circ}$
(28) The magnitude of two forces acting at a point $\mathrm{F}, \sqrt{3} \mathrm{~F}$ newton. If the magnitude of their resultant is 2 F newton, then the measure of their included angle equals $\qquad$
(a) $30^{\circ}$
(b) $60^{\circ}$
(c) $90^{\circ}$
(d) $120^{\circ}$
(29) If $\stackrel{\rightharpoonup}{\mathrm{R}}=\overrightarrow{\mathrm{F}}_{1}+\overrightarrow{\mathrm{F}}_{2}$ and $\|\stackrel{\rightharpoonup}{\mathrm{R}}\|=\left\|\overrightarrow{\mathrm{F}}_{1}\right\|-\left\|\overrightarrow{\mathrm{F}_{2}}\right\|$, then the measure of the angle between $\overrightarrow{\mathrm{F}_{1}}, \stackrel{\rightharpoonup}{\mathrm{~F}_{2}}$ equals $\qquad$
(a) zero
(b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$
(d) $\pi$
(30) If the magnitude of the resultant of two forces act at a point is maximum value , then the measure of the angle between the two forces equal $\qquad$
(a) $180^{\circ}$
(b) $120^{\circ}$
(c) zero
(d) $60^{\circ}$
(31) The measure of the angle between $\vec{F}_{1}$ and the resultant of the two forces $\left(\overrightarrow{F_{1}}+\vec{F}_{2}\right)$ and $\left(\overrightarrow{F_{1}}-\overrightarrow{\mathrm{F}_{2}}\right)$ is $\qquad$
(a) zero
(b) $\pi$
(c) $\frac{\pi}{2}$
(d) $\frac{\pi}{3}$
(32) If $\overrightarrow{\mathrm{R}_{1}}$ is the resultant of the two forces $\left(\overrightarrow{\mathrm{F}_{1}}, \overrightarrow{\mathrm{~F}_{2}}\right)$ and $\overrightarrow{\mathrm{R}_{2}}$ is the resultant of the two forces $\left(\overrightarrow{\mathrm{F}_{1}},-\overrightarrow{\mathrm{F}_{2}}\right),\left\|\overrightarrow{\mathrm{F}_{1}}\right\|=\left\|\overrightarrow{\mathrm{F}_{2}}\right\|$, then
(a) $\overrightarrow{R_{1}} \perp \overrightarrow{R_{2}}$
(b) $\overrightarrow{\mathrm{R}_{1}}=\overrightarrow{\mathrm{R}_{2}}$
(c) $\left\|\overrightarrow{\mathrm{R}_{1}}\right\|=\left\|\overrightarrow{\mathrm{R}_{2}}\right\|$
(d) $\stackrel{\rightharpoonup}{R_{1}} / / \overrightarrow{R_{2}}$
(33) Two forces of magnitudes 4 and 6 newton. The measure of the angle between them is $90^{\circ}$, then the tangent of the angle between the resultant and the first force equal $\qquad$ ....
(a) $\frac{2}{3}$
(b) $\frac{3}{2}$
(c) $2 \sqrt{13}$
(d) $\frac{\sqrt{6}}{2}$
(34) The magnitudes of two perpendicular forces are 6,8 newton then the measure of the angle between the resultant and the first force is $\qquad$
(a) $\sin ^{-1} \frac{4}{3}$
(b) $\cos ^{-1} \frac{4}{3}$
(c) $\tan ^{-1} \frac{4}{3}$
(d) $\tan ^{-1} \frac{3}{4}$
(35) Two forces of magnitudes $\mathrm{F}, 2 \mathrm{~F}$ newton act at a point, if the resultant of them is perpendicular to one of them, then $\mathrm{R}=$ $\qquad$
(a) $\sqrt{5} \mathrm{~F}$
(b) $\sqrt{3} \mathrm{~F}$
(c) 3 F
(d) F
(36) Two forces of magnitudes $3 \sqrt{2}$ and 6 newton and the measure of the angle between them is $135^{\circ}$, then the measure of the angle between their resultant and the second force is $\qquad$
(a) $30^{\circ}$
(b) $45^{\circ}$
(c) $60^{\circ}$
(d) $90^{\circ}$
(37) Two forces of magnitudes 12,15 newton act at a particle and the measure of the enclosing angle between them is $\theta^{\circ}$, where $\cos \theta=\frac{-4}{5}$, then the measure of the included angle between the resultant and the first force $=$ $\qquad$ ..
(a) zero
(b) 30
(c) 90
(d) $36^{\circ} 5 \grave{2}$
(38) The magnitude of two forces acting on a particle are 5,8 newton, then the smallest value of their resultant $=$ $\qquad$ newton.
(a) 2
(b) 3
(c) 7
(d) 13
(39) Two forces of magnitudes 9 newton, 1000 dyne, the maximum value of their resultant $\qquad$
(a) 1009 dyne.
(b) 1009 newton.
(c) 9.01 dyne.
(d) 9.01 newton.
(40) Two forces of magnitudes $5, \mathrm{~F}$ newton, if the smallest resultant of them is 10 newton, $\mathrm{F}>5$, then $\mathrm{F}=$ $\qquad$ newton.
(a) 6
(b) 10
(c) 15
(d) 20
(41) Two forces act at a point. The magnitude of the two forces are $5 \mathrm{~F}, 3 \mathrm{~F}$. If the maximum value of their resultant is 40 newton, then the minimum value of their resultant $\qquad$ newton.
(a) 10
(b) 20
(c) 5
(d) zero
(42) Two forces act at a point. The magnitudes of the two forces are 5,3 newton, then the magnitude of their resultant measure by newton $\in$. $\qquad$ ...
(a) $[2,8]$
(b) $] 2,8[$
(c) $[3,5]$
(d) $] 3,5[$
(43) If $\theta$ is the angle between two forces of magnitudes 2 newton, 6 newton $, \theta \in] 0, \pi]$, then the magnitude of their resultant measured by newton $\in$
(a) $] 4,8[$
(b) $[4,8[$
(c) $] 4,8]$
(d) $[4,8]$
(44) Two forces of equal magnitude and the magnitude of their resultant equal 16 newton when the measure of the angle between the two forces is $\frac{\pi}{2}$, then the maximum value of their resultant equal $\qquad$ newton.
(a) 32
(b) $8 \sqrt{2}$
(c) $16 \sqrt{2}$
(d) zero
(45) Two forces of magnitude $F_{1}, F_{2}$ kg.wt., where $F_{1}>F_{2}$ and the magnitude of smallest and greatest resultant of them are 3 and $12 \mathrm{gm} . \mathrm{wt}$. respectively , then $\mathrm{F}_{1}^{2}-\mathrm{F}_{2}^{2}=$ $\qquad$ ...
(a) 12
(b) 3
(c) 9
(d) 36
(46) The magnitude of two forces are 12,17 newton then the difference between the greatest and the smallest value of their resultant $=$ $\qquad$ newton.
(a) 29
(b) 5
(c) 14
(d) 24
(47) Two forces of magnitude $F, \sqrt{3} \mathrm{~F}$ newton meeting at a point and the magnitude of their resultant is $R_{1}$ when the measure of the angle between the two forces is $90^{\circ}$ , and their resultant becomes $\mathrm{R}_{2}$ when the measure of the angle between the two forces is $150^{\circ}$, then
(a) $R_{1}=R_{2}$
(b) $\mathrm{R}_{1}=2 \mathrm{R}_{2}$
(c) $\mathrm{R}_{1}=\frac{3}{5} \mathrm{R}_{2}$
(d) $R_{1}=\frac{1}{2} R_{2}$
(48) The direction of the resultant of the forces which represented in the opposite figure is $\qquad$
(a) $\overrightarrow{\mathrm{OX}}$
(b) $\overrightarrow{O X}$
(c) $\overrightarrow{\mathrm{Oy}}$
(d) $\overline{\mathrm{Oy}}$

(49) Two forces act at a point and the magnitude of smallest and greatest resultant of them are 0 and 12 newton respectively, then
(a) magnitude of one force is three times magnitude of the other.
(b) magnitude of one force is twice magnitude of the other.
(c) the two forces are equal in magnitude.
(d) the two forces are perpendicular.

## Second Essay questions

1 Find the magnitude and the direction of the resultant of two perpendicular forces of magnitudes 8 and 15 kg .wt. acting at a particle.

2 The magnitude of the resultant of two perpendicular forces is 50 newton. If the resultant makes with the first force an angle of measure $30^{\circ}$, find the magnitude of each of these two forces.
« $25 \sqrt{3}, 25$ newton "
3 Two forces of magnitudes 30 and 16 newton act at a particle, if the magnitude of their resultant is 26 newton. Find the measure of the angle between these two forces. « $120^{\circ}$ "

4 Two forces are of magnitudes 9 and 6 kg .wt. act at a particle. The measure of the included angle is $\alpha$, find $\alpha$ if the magnitude of the resultant is $3 \sqrt{7} \mathrm{~kg}$.wt., find the measure of the angle between the resultant and the great force. $\quad \alpha \alpha=120^{\circ}, \theta=40^{\circ} 5 \grave{3} 36^{\approx}$ "
5. Two forces acted at a point. If the magnitude of the first is $15 \mathrm{~kg} . w t$. towards East and the second is of magnitude $18 \mathrm{~kg} . \mathrm{wt}$. in the direction $30^{\circ}$ West of the North. Calculate the magnitude and the direction of the resultant.

6 Two forces of magnitudes 12 , F kg.wt. act on a point. The first force acts in direction of East and the second force acts in direction $60^{\circ}$ South of the West. Find the magnitude of F and the magnitude of the resultant if it is known that the line of action of the resultant acts in the direction $30^{\circ}$ South of the East. $\quad 6 \mathrm{~kg} . \mathrm{wt}, 96 \sqrt{3} \mathrm{~kg} . \mathrm{wt}$.»
1 Two forces act at a particle and they include an angle of measure $\alpha$ where $\tan \alpha=\frac{-1}{\sqrt{3}}$ If the resultant is perpendicular to the small force and the magnitude of the great force equals 30 kg .wt. What is the magnitude of each of the small force and the resultant ?

8 Find the magnitude and the direction of the resultant in each of the following figures :
(1)

(2)

(3)


9 Dan forces of magnitudes $\mathrm{F}, 4$ newton act on a particle and the measure of the angle between their directions is $120^{\circ}$, the magnitude of their resultant equals $4 \sqrt{3}$ newton. Find the magnitude of $\overrightarrow{\mathrm{F}}$ and the measure of the angle that $\overrightarrow{\mathrm{R}}$ from with $\overrightarrow{\mathrm{F}}$ «8 newton, $30^{\circ}$ "

10 Two forces of magnitudes $\sqrt{3} \mathrm{~F}$ and 2 F act at a point. Find the measure of the angle included between them if their resultant is perpendicular to the small force and if $\mathrm{F}=15$ Find the magnitude of the resultant.

11 Two forces of magnitudes $2 \sqrt{2}$ and $F$ newton act at a particle and the magnitude of their resultant is $\sqrt{2}$ newton. If the resultant is perpendicular to the second force, find F and the measure of the angle between the two forces.

12 Two forces of magnitudes 16 and F kg.wt. act on a particle and the measure of the angle between them is $120^{\circ}$. If their resultant is inclined to the force $16 \mathrm{~kg} . \mathrm{wt}$. by an angle whose measure is $30^{\circ}$, find the magnitude of F and the resultant.

18 Three forces of magnitudes $5,10,4 \sqrt{7} \mathrm{~N}$. act on a particle, if the measure of the angle between the first and the second forces equals $60^{\circ}$, find the magnitude of the maximum and the minimum resultant for the three forces. «9 $\sqrt{7}$ newton, $\sqrt{7}$ newton»

14 Two forces of magnitudes 2 F and 3 F newton. The angle between them is of measure $\theta$ , find the value of $\theta$ if the magnitude of their resultant is :
(1) 3 F
(2) F
(3) 5 F
(4) $\sqrt{13} \mathrm{~F}$

15 Two forces of magnitudes $2, \mathrm{~F}$ newton, the angle between them is of measure $120^{\circ}$ Find F in each of the two cases :
(1) The direction of the resultant is perpendicular to the second force.
(2) The resultant inclines by $45^{\circ}$ to the $2^{\text {nd }}$ force.
«1, $\sqrt{3}+1$ newton »
$10 \mathrm{~F}_{1}$ and $\mathrm{F}_{2}$ newton are magnitudes of two forces intersect at a point and their resultant equals $R$ newton where $R \in[2,10], F_{1}>F_{2}$, find each of $F_{1}$ and $F_{2}$, then find $R$ when the measure of the angle between them is $120^{\circ}$
« $6,4,2 \sqrt{7}$ newton »
11 Two forces act at a point, the value of one is 3 N . more than the other.
If the magnitude of their resultant is $3 \sqrt{3}$ newton and is perpendicular to the smaller force. Find the magnitude of each force and the measure of the angle between them.

18 The resultant of two forces $F_{1}$ and $F_{2}$ is $\sqrt{10}$ newton when $F_{1} \perp F_{2}$ and their resultant becomes $\sqrt{13}$ newton when the angle between $F_{1}$ and $F_{2}$ becomes $60^{\circ}$, find $F_{1}$ and $F_{2}$

19 Two forces of equal magnitude meeting at a point and the magnitude of their resultant equals 12 kg .wt. if the direction of one of them is reversed then the magnitude of the resultant becomes 6 kg .wt. Find the magnitude of each force.

20 Two forces $\overrightarrow{\mathrm{F}_{1}}, \overrightarrow{\mathrm{~F}_{2}}$ meet at a point. Their resultant is R gm.wt. The angle between them is of measure $120^{\circ}$. If the direction of $\overrightarrow{\mathrm{F}}_{2}$ is reversed, the resultant will be $\mathrm{R} \sqrt{3} \mathrm{gm}$.wt., prove that $\mathrm{F}_{1}=\mathrm{F}_{2}$ and the resultant in the first case is perpendicular to the second case.
$214, \mathrm{~F}$ are two forces acting at a point and their resultant is 10 newton and makes an angle of measure $60^{\circ}$ with the force 4 newton. Find the value of F .

22 The difference between the magnitudes of two forces acting at a point is 15 newton. and their resultant $=35$ newton in magnitude when the measure of the angle between the two forces $=120^{\circ}$, find the magnitude of each of the two forces.

28 The sum of magnitudes of two forces is 4 newton when the measure of the angle between them is $60^{\circ}$, then the resultant becomes $\sqrt{13}$ newton. Find the magnitude of each of the two forces.
«1,3 newton»
24 The sum of magnitudes of two forces acting at a point is $40 \mathrm{~kg} . \mathrm{wt}$. the magnitude of their resultant is 20 kg .wt. and it is perpendicular to the smaller force. Find the magnitude of each of the two forces and the cosine of the angle between them. «15, $25 \mathrm{~kg} . \mathrm{wt} .,-\frac{3}{5}$ "

25 (1) Two forces of same magnitude F kg.wt. enclose between them an angle of measure $120^{\circ}$. If the two forces are doubled and the measure of the angle between them became $60^{\circ}$, then the magnitude of their resultant increases by $11 \mathrm{~kg} . w t$. , than the first case. Find the magnitude of F
$261 \mathrm{~F}, 2 \mathrm{~F}$ are two forces act on a particle and enclose between them an angle of measure $\alpha$ The magnitude of their resultant equals $\sqrt{5} \mathrm{~F}(\mathrm{~m}+1)$ and if the measure of the angle between them becomes $\left(90^{\circ}-\alpha\right)$, then the magnitude of the resultant will be $\sqrt{5} \mathrm{~F}(\mathrm{~m}-1)$
Prove that $: \tan \alpha=\frac{m-2}{m+2}$

## Third Higher skills

1 Choose the correct answer from those given :
(1) If the ratio between the maximum and the minimum values of the resultant of two forces is $7: 3$, then the ratio between the two forces $=$ $\qquad$
(a) $7: 4$
(b) $7: 3$
(c) $5: 3$
(d) $5: 2$
(2) If the ratio among magnitudes of two forces and their resultant is $4: 3: \sqrt{13}$ respectively, then the measure of the angle between the two forces $=$ $\qquad$
(a) $30^{\circ}$
(b) $60^{\circ}$
(c) $90^{\circ}$
(d) $120^{\circ}$
(3) If the resultant of two forces $\overrightarrow{\mathrm{F}_{1}}, \overrightarrow{\mathrm{~F}_{2}}$ is perpendicular on $\overrightarrow{\mathrm{F}_{1}}$, then the measure of the angle between the two forces $\overrightarrow{\mathrm{F}}_{1}, \overrightarrow{\mathrm{~F}}_{2}$ equals $\qquad$
(a) $\cos ^{-1}\left(\frac{F_{1}}{F_{2}}\right)$
(b) $\cos ^{-1}\left(\frac{-\mathrm{F}_{1}}{\mathrm{~F}_{2}}\right)$
(c) $\sin ^{-1}\left(\frac{F_{1}}{F_{2}}\right)$
(d) $\sin ^{-1}\left(\frac{-F_{1}}{\mathrm{~F}_{2}}\right)$
(4) If the resultant of two perpendicular forces makes an angle of measure $\theta$ to the greater force which of the following values could be a value of $\theta$ ?
(a) $90^{\circ}$
(b) $70^{\circ}$
(c) $45^{\circ}$
(d) $10^{\circ}$
(5) $\vec{F}_{1}, \overrightarrow{F_{2}}$ are two forces acting at a point and their resultant is R. If $\overrightarrow{F_{2}}$ reversed then their resultant rotates with angle of measure $90^{\circ}$, then $\qquad$
(a) $\mathrm{F}_{1}=\mathrm{F}_{2}$
(b) $\mathrm{F}_{1}=2 \mathrm{~F}_{2}$
(c) $\mathrm{F}_{1}=\frac{1}{2} \mathrm{~F}_{2}$
(d) nothing of the previous.
(6) The magnitudes of two forces acting at a point are $4, F$ newton and the measure of their included angle is $120^{\circ}$, then F which makes the resultant minimum equals $\qquad$ newton.
(a) 1
(b) 2
(c) 3
(d) 4
( 7 ) If $\theta_{1}$ is the measure of the angle between the resultant of two forces $\left(\vec{F}_{1}, \overrightarrow{F_{2}}\right)$ and the force $\vec{F}_{1}$ and $\theta_{2}$ is the measure of the angle between the resultant of the two forces $\left(\vec{F}_{1}, 2 \vec{F}_{2}\right)$ and the force $\vec{F}_{1}$, then $\qquad$
(a) $\theta_{1}=\theta_{2}$
(b) $\theta_{1}>\theta_{2}$
(c) $\theta_{1}<\theta_{2}$
(d) $\theta_{1}+\theta_{2}=\frac{\pi}{2}$
(8) The magnitudes of two forces acting at a point are $\mathrm{F}, \sqrt{3} \mathrm{~F}$ newton and the magnitude of their resultant is F newton and $\theta_{1}$ is the measure of the angle between $F, R$ and $\theta_{2}$ is the measure between $\sqrt{3} F$ and $R$, then $\qquad$
(a) $\theta_{1}=\theta_{2}$
(b) $\theta_{1}=\frac{1}{2} \theta_{2}$
(c) $\theta_{1}=3 \theta$
(d) $\theta_{1}=4 \theta_{2}$
(9) The magnitudes of two forces acting at a point are $F_{1}, F_{2}$ where : $3 \leq F_{1} \leq 12$ , $4 \leq \mathrm{F}_{2} \leq 16$ and the magnitude of their resultant is R and the measure of their included angle is $90^{\circ}$, then $\qquad$
(a) $5 \leq \mathrm{R} \leq 20$
(b) $7 \leq \mathrm{R} \leq 28$
(c) $0 \leq \mathrm{R} \leq 18$
(d) $1 \leq \mathrm{R} \leq 4$
(10) Two forces meet at a point, their magnitudes are $\mathrm{F}_{1}, \mathrm{~F}_{2}$ where $1 \leq \mathrm{F}_{1} \leq 9,3 \leq \mathrm{F}_{2} \leq 7$ and the magnitude of their resultant $R$, then $\qquad$
(a) $2 \leq R \leq 16$
(b) $4 \leq \mathrm{R} \leq 16$
(c) $6 \leq R \leq 16$
(d) $0 \leq R \leq 16$
(11) The magnitudes of two forces acting at a point are $\mathrm{F}_{1}, \mathrm{~F}_{2}$ where $5 \leq \mathrm{F}_{1} \leq 20$ , $12 \leq \mathrm{F}_{2} \leq 21$ and the magnitude of their resultant is R , the measure of the angle between them is $\theta$ where $0 \leq \theta \leq \frac{\pi}{2}$ then $\qquad$
(a) $13 \leq \mathrm{R} \leq 29$
(b) $0 \leq \mathrm{R} \leq 41$
(c) $13 \leq \mathrm{R} \leq 41$
(d) $17 \leq \mathrm{R} \leq 29$

2 One of two forces is half the other in magnitude, they have a certain resultant. If the small force increased by 4 kg .wt. and the great force becomes double, then their resultant stays in the same direction of the first case, find the magnitudes of the two forces and the ratio between the magnitudes of the two resultants in the two cases. « $4,8 \mathrm{~kg} . \mathrm{wt} ., 1: 2$ »
$3 \vec{F}_{1}$ and $\overrightarrow{F_{2}}$ are two forces meeting at a point and their resultant is $R$ newton. If the direction of $\overrightarrow{\mathrm{F}_{2}}$ becomes in the opposite direction, then the magnitude of the resultant becomes $\mathrm{R} \sqrt{3}$ newton and the resultant becomes perpendicular to the first resultant. Find the measure of the angle between the two forces.


From the school book

## First

Multiple choice questions

Choose the correct answer from the given ones :
(1) In the opposite figure:

If the force of magnitude 10 N . is resolved into two components $\overrightarrow{\mathrm{F}}_{1}$ and $\overrightarrow{\mathrm{F}}_{2}$ inclined to

the force by two angles of measures $60^{\circ}$ and $90^{\circ}$ respectively,
then $\mathrm{F}_{2}=$ $\qquad$ N.
(a) $5 \sqrt{3}$
(b) 10
(c) $10 \sqrt{3}$
(d) 20
( 2 ) In the opposite figure:
If the force of magnitude 12 N . is resolved into two components $\overrightarrow{\mathrm{F}_{1}}$ and $\overrightarrow{\mathrm{F}_{2}}$ inclined to the force by two angles of measures $30^{\circ}$ and $90^{\circ}$
respectively, then $\mathrm{F}_{2}=$ $\qquad$ N .

(a) 10
(b) $10 \sqrt{3}$
(c) $6 \sqrt{3}$
(d) $4 \sqrt{3}$

## ( 3 ) In the opposite figure :

If the force of magnitude 12 N . is resolved into two components $\stackrel{\rightharpoonup}{\mathrm{F}_{1}}$ and $\stackrel{\rightharpoonup}{\mathrm{F}_{2}}$, then $\mathrm{F}_{1}=$ $\qquad$ newton.
(a) $12 \cos 75^{\circ}$
(b) $12 \cos 45^{\circ}$
(c) $6 \csc 45^{\circ}$
(d) $6 \csc 75^{\circ}$

(4) In the opposite figure :

If the force of magnitude 50 newton is resolved into two components $\overrightarrow{\mathrm{F}_{1}}$ and $\overrightarrow{\mathrm{F}_{2}}$, then $\mathrm{F}_{1}+\mathrm{F}_{2}=$ $\qquad$ newton.
(a) 50
(b) 25
(c) $50 \sqrt{2}$
(d) $50 \sqrt{3}$

( 5 ) In the opposite figure :
If the force $\stackrel{\rightharpoonup}{F}$ is resolved into the two perpendicular components $\overrightarrow{\mathrm{F}_{1}}$ and $\overrightarrow{\mathrm{F}_{2}}$, the vector of the force $\stackrel{\rightharpoonup}{\mathrm{F}}$ bisects the angle between the directions of $\vec{F}_{1}$ and $\overrightarrow{\mathrm{F}}_{2}$ and $\left\|\overrightarrow{\mathrm{F}}_{1}\right\|=6 \sqrt{2}$ newton , then $\|\stackrel{\rightharpoonup}{\mathrm{F}}\|=$ $\qquad$ newton.

(a) 6
(b) $6 \sqrt{2}$
(c) 12
(d) $12 \sqrt{2}$
( 6 ) In the opposite figure :
If the force of magnitude 100 newton is resolved into two forces $\vec{F}_{1}$ and $\vec{F}_{2}$ and the force is measured by newton , then $\left(\mathrm{F}_{1}, \mathrm{~F}_{2}\right)=$ $\qquad$
(a) $(50,50 \sqrt{3})$
(b) $(50 \sqrt{3}, 10)$
(c) $(50,50)$
(d) $(10,10)$


## ( 7 ) In the opposite figure :

A force of magnitude 20 newton. acts in the direction $30^{\circ}$ North of the East is resolved into two perpendicular components, then the magnitude of the component in North direction $=$ $\qquad$ newton.
(a) $10 \sqrt{3}$
(b) 20
(c) 10
(d) 5


## (8) In the opposite figure :

A force of magnitude $20 \sqrt{2} \mathrm{~kg}$.wt. acts in the Western North direction, is resolved into two component. One of them of magnitude $F_{1}$ in the Eastern North direction and the other of magnitude $\mathrm{F}_{2}$ in the direction of West , then $\mathrm{F}_{2}=$ $\qquad$ kg.wt.
(a) 30
(b) 40
(c) 50
(d) $40 \sqrt{2}$

(9) In the opposite figure:

If a force $\vec{F}$ is resolved into two components in the directions of the coordinate axes, then the magnitude of the component of this force in the direction of $\overrightarrow{\mathrm{O}}$ equals $\qquad$ newton.
(a) 10
(b) 6
(c) 8
(d) $\frac{40}{3}$

$x$
(10) A force of magnitude $10 \sqrt{2}$ gm.wt. acts in the Eastern South direction, is resolved into two perpendicular components, then the magnitude of the component in the South direction $=$ $\qquad$ gm.wt.
(a) 5
(b) 10
(c) $10 \sqrt{2}$
(d) $5 \sqrt{2}$
(11) A force of magnitude 6 newton acts in direction of North. It is resolved into two perpendicular components, so its component in direction of the East of magnitude $\qquad$ newton.
(a) zero
(b) 3
(c) $3 \sqrt{2}$
(d) 6
(12) A force of magnitude $4 \sqrt{2}$ newton acts in direction of East. It is resolved into two perpendicular components, so its component in the direction of Northern East of magnitude $\qquad$ newton.
(a) zero
(b) $4 \sqrt{2}$
(c) 4
(d) 6
(13) The magnitude of a force is 6 newton and acts towards the North. It is resolved into two perpendicular components then its component in direction of Eastern North of magnitude $\qquad$ newton.
(a) 6
(b) $3 \sqrt{2}$
(c) $2 \sqrt{3}$
(d) zero
(14) A force of magnitude $5 \sqrt{3}$ newton acts in the direction $30^{\circ}$ East of the North, is resolved into two perpendicular components, then the magnitude of its component in the East direction $=$ $\qquad$ newton.
(a) $\frac{5 \sqrt{3}}{2}$
(b) $\frac{15}{2}$
(c) $\frac{15 \sqrt{3}}{2}$
(d) $15 \sqrt{3}$
(15) The magnitude of a force is 8 newton and acts in East direction. It is resolved into two components, the angle between the two components is $120^{\circ}$, then its component in South direction $=$ newton.
(a) 16
(b) 8
(c) $8 \sqrt{3}$
(d) $\frac{8 \sqrt{3}}{3}$
(16) A force of magnitude 40 newton acts vertically upwards is resolved into two components one of them is horizontal of magnitude 20 newton, then the magnitude of the other $=$ $\qquad$ newton.
(a) 20
(b) $20 \sqrt{3}$
(c) $20 \sqrt{5}$
(d) $10 \sqrt{3}$
(17) Force of magnitude $F$ newton is resolved into two components $\vec{F}_{1}$ and $\overrightarrow{F_{2}}$ and they make angles of measure $60^{\circ}, 90^{\circ}$ respectively but on different sides from the line of action of $\vec{F}$, then $F_{1}=$ $\qquad$
(a) $2 \mathrm{~F}_{2}$
(b) $\frac{\sqrt{3}}{2} \mathrm{~F}_{2}$
(c) $\frac{2}{\sqrt{3}} \mathrm{~F}_{2}$
(d) $\frac{1}{2} \mathrm{~F}_{2}$
(18) In the opposite figure :

A vertical force of magnitude 75 newton is resolved into two components, one of them is horizontal of magnitude $F_{1}$ and the other is of magnitude $F_{2}$
 , then $\mathrm{F}_{2}=$ $\qquad$ newton.
(a) 75
(b) $75 \sqrt{3}$
(c) 150
(d) $150 \sqrt{3}$
(19) In the opposite figure :

The force $\stackrel{\rightharpoonup}{F}$ is the resultant of the two forces $\overrightarrow{\mathrm{F}_{1}}, \overrightarrow{\mathrm{~F}_{2}}$, then $\frac{\mathrm{F}_{1}+\mathrm{F}_{2}}{\mathrm{~F}}=$
(a) $\sin 30^{\circ}+\sin 45^{\circ}$
(b) $\frac{\sin 75^{\circ}+\sin 30^{\circ}}{\sin 75^{\circ}}$
(c) $\frac{\sin 45^{\circ}+\sin 30^{\circ}}{\sin 75^{\circ}}$
(d) $\frac{\sin 75^{\circ}}{\sin 30^{\circ}}+\frac{\sin 75^{\circ}}{\sin 45^{\circ}}$

(20) $A B C D E F$ is a regular hexagon. A force of magnitude 20 newton acts in direction of $\overrightarrow{\mathrm{AD}}$, then the magnitudes of the components of the force in direction of $\overrightarrow{\mathrm{AC}}, \overrightarrow{\mathrm{AF}}$ respectively are $\qquad$
(a) $10 \sqrt{3}, 10$
(b) $5 \sqrt{3}, 10$
(c) $10,10 \sqrt{3}$
(d) $20 \sqrt{3}, 20$
(21) In the opposite figure :

The force $\vec{F}$ has been resolved into two components $\vec{F}_{1}, \stackrel{\rightharpoonup}{F_{2}}$, then $\frac{\mathrm{F}_{1}}{\mathrm{~F}_{2}}=$
(a) $\frac{\sin \theta_{2}}{\sin \theta_{1}}$
(b) $\sin \left(\frac{\theta_{2}}{\theta_{1}}\right)$
(c) $\sin \left(\theta_{1}+\theta_{2}\right)$
(d) $\frac{\sin \theta_{1}}{\sin \theta_{2}}$


## (22) In the opposite figure :

$A B C D E F$ is a regular hexagon. Force of magnitude 15 N . acts along $\overrightarrow{\mathrm{AC}}$ and it has been resolved into two components $\overrightarrow{\mathrm{F}_{1}}$ and $\overrightarrow{\mathrm{F}_{2}}$ as shown in the figure $\mathrm{F}_{1}: \mathrm{F}_{2}=$
(a) $\sqrt{3}: 2$
(b) $2: 1$
(c) $1: 2$
(d) $1: \sqrt{3}$

(23) In the opposite figure :

If a body of weight 10 newtons is placed on a smooth plane inclined to the horizontal at an angle of measure $30^{\circ}$, then the component of the weight in direction of line of the greatest slope downward $=$ $\qquad$ N .

(a) $5 \sqrt{2}$
(b) $5 \sqrt{3}$
(c) 5
(d) $10 \sqrt{3}$
(24) If a body of weight ( W ) is placed on a smooth plane inclined to horizontal by angle ( $\theta$ ) , so the component of its weight in direction of the plane equals $\qquad$ ...
(a) W
(b) $\mathrm{W} \sin \theta$
(c) $\mathrm{W} \cos \theta$
(d) $\mathrm{W} \tan \theta$
(25) If a body of weight (W) is placed on an inclined smooth plane makes an angle of measure $(\theta)$ with the horizontal, then its weight component in the perpendicular direction of the plane is
(a) $\mathrm{W} \sin \theta$
(b) $\mathrm{W} \cos \theta$
(c) $\mathrm{W} \tan \theta$
(d) $\mathrm{W} \csc \theta$
(26) If a body of weight (W) is placed on an inclined smooth plane makes an angle of measure $(\theta)$ with the vertical, then its weight component in direction of the plane is $\qquad$
(a) $\mathrm{W} \sin \theta$
(b) $\mathrm{W} \cos \theta$
(c) W
(d) $\mathrm{W} \tan \theta$
(27) A body of weight (W) newton is placed on an inclined plane makes an angle of measure $(\theta)$ with the horizontal, then the components of its weight in direction line of greatest slope and its perpendicular are 7,24 newton respectively, then the magnitude of the weight $(\mathrm{W})=$ $\qquad$ newton.
(a) 7
(b) 24
(c) 25
(d) 31
(28) A tractor drags a car with a force 1200 newtons. It's required to replace the tractor by another two tractors at B and C attached with two cables to the car and the angle between the two cables is $90^{\circ}$. If one of the two cables inclined to the tractor A at an angle $60^{\circ}$, then the tensions in the two cables B and C
 are $\qquad$ newtons.
(a) 600,600
(b) 800,400
(c) $600 \sqrt{3}, 600$
(d) 700,500
(29) A truck has broken down traffic officers try to pull the truck by using two draging cars. The resultant of their tensions is a horizontal tension of magnitude 6000 newtons as shown in the figure then $T_{2}=\cdots \cdots \cdots \cdots$ to the nearest newton.

(a) 3105
(b) 3606
(c) 4392
(d) 4293
(30) In the opposite figure :

A body of weight (W) newtons is placed on a plane inclined to the horizontal at an angle of measure $(\theta)$. It is tied by a light string $\overline{\mathrm{BC}}$ inclined to the plane at an angle of measure $20^{\circ}$ above the plane. $\mathrm{F}_{1}$ and $\mathrm{F}_{2}$ are the components of the tension in direction of the plane and
 perpendicular to the plane then.........
(a) $\mathrm{F}_{2}=\mathrm{T} \cos \theta$
(b) $\mathrm{F}_{1}=\mathrm{T} \sin \left(20^{\circ}+\theta\right)$
(c) $\mathrm{F}_{1}=\mathrm{T} \cos \left(20^{\circ}+\theta\right)$
(d) $\mathrm{T}=\mathrm{F}_{1} \sec 20^{\circ}$

## Second Essay questions

1 A force of magnitude 600 kg . wt. acts on a particle. Find its two components in two directions making with the force two angles of measures $30^{\circ}$ and $45^{\circ}$ « $439.23,310.68$ gm.wt. "

2 A force of magnitude 100 gm.wt. acts in the direction of Western North. Find its components in the North direction and in West direction.

3 A force of magnitude 12 kg . wt. acting in the direction of Eastern North was resolved into two components. One in the direction of East and the other in the direction of Western North. Find these two components.

4 Resolve a horizontal force of magnitude 160 gm.wt. in two perpendicular directions. One of them inclined to the horizontal with an angle of measure $30^{\circ}$ upwards.
$\qquad$
5. A force of magnitude 300 dyne. acts in the North direction. Find the magnitudes of the two perpendicular components if one of them acts in the direction $30^{\circ}$ North of East.

6 A force of magnitude 18 newton acts in the direction of South. Find its two components in the two directions $60^{\circ}$ East of the South and the other direction towards $30^{\circ}$ West of the South.

1 Resolve a force of magnitude 90 newton into two equal forces in magnitude and the measure of the angle between their lines of action is $60^{\circ}$
« $30 \sqrt{3}$ newton »

8 A body of weight 80 newton is placed on a horizontal plane. Find the two perpendicular components of the weight if one of them inclines to the horizontal with $30^{\circ}$ downwards.

9 Two forces act at a point. $\alpha$ is the angle between them and $\tan \alpha=-\frac{1}{\sqrt{3}}$, If their resultant is perpendicular to the smaller force and the greater force 30 newton. Find the magnitude of the other force and the resultant.

10 Resolve a force of magnitude F newton in the North direction into two components, the first in the direction $30^{\circ}$ North of East with magnitude 40 newton and the other is in the West direction. Find each of the magnitude of the force $F$ and the magnitude of the other component.

11 DC. A rigid body of weight 42 netwon is placed on a plane inclined to the horizontal with an angle of measure $60^{\circ}$. Find the two components of the weight of the body in the direction of the line of the greatest slope and the direction normal to it. \& $21 \sqrt{3}, 21$ newton »

12 A body of weight 60 newton is placed on an inclined plane, at an angle of measure $\theta$ where $\tan \theta=\frac{3}{4}$, find the magnitudes of the two components of the weight in the direction of the line of greatest slope of the plane and the perpendicular to it.

## 13 In the opposite figure :

Resolve the vertical force of magnitude 120 gm.wt. into two components, one of them in the horizontal direction and the other inclined by an angle of measure $48^{\circ}$ with the line of action of the force.


* $133.27,179.34$ gm.wt. *

14 The opposite figure represents an angle of a bridge, the force $\overrightarrow{\mathrm{F}}$ of magnitude 30 newton is resolved into two perpendicular components, the magnitude of one of them is $15 \sqrt{3}$ newton Find the magnitude of the other component.


[^0]
## 15 In the opposite figure :

A lamp of weight 20 newton suspended by two metal rods $\overline{\mathrm{AC}}, \overline{\mathrm{BC}}$ inclined to the horizontal by two equal angles, the measure of each is $5^{\circ}$ :
(1) Resolve the weight of the lamp into two components in the

(20) directions $\overrightarrow{\mathrm{AC}}, \overrightarrow{\mathrm{BC}}$ approximating the result to the nearest netwon.
(2) What happens to the magnitude of the components of the weight in the directions of the two metal rods if the measure of the inclination angle to the horizontal decreased to be smaller than $5^{\circ}$ ? And what do you expect to the components when the rods become horizontal ? Justify your answer. «114.74, 114.74 newton »

16 An inclined plane of length 130 cm . and height 50 cm . a rigid body of weight $390 \mathrm{gm} . \mathrm{wt}$. is placed on it. Find the two components of the weight in the direction of the line of greatest slope of the plane and the perpendicular to it.

## 11 Cl In the opposite figure :

A cruiser is pulled by two ships B and C using two strands hanged to a point A on the cruiser, the measure of the angle between the two strands equals $75^{\circ}$, if the measure of the angle between
 one of the strands and $\overrightarrow{\mathrm{AD}}$ equals $45^{\circ}$ and the resultant of the forces used to pull the cruiser equals 5000 newton and acts on $\overrightarrow{\mathrm{AD}}$
Find the tension in the two strands.
$2588.2,3660.3$ newton »

# Exercise 

## The resultant of coplanar forces meeting at a point

10 From the school book


## Choose the correct answer from those given :

(where $\vec{i}$ and $\vec{j}$ are the two fundamental unit vectors in two perpendicular directions)
(1) If $\overrightarrow{F_{1}}=\vec{i}-\vec{j}, \overrightarrow{F_{2}}=2 \vec{i}-4 \vec{j}, \vec{R}=2 a \vec{i}-3 b \vec{j}$, then $a+b=$ $\qquad$
(a) 3
(b) $3 \frac{1}{3}$
(c) $3 \frac{1}{6}$
(d) 12
(2) If $\overrightarrow{F_{1}}=3 \vec{i}-2 \vec{j}, \overrightarrow{F_{2}}=a \vec{i}-\vec{j}, \overrightarrow{F_{3}}=4 \vec{i}-b \vec{j}, \vec{R}=6 \vec{i}-4 \vec{j}$ , then $(\mathrm{a}, \mathrm{b})=$
(a) $(1,-1)$
(b) $(-1,1)$
(c) $(-1,-1)$
(d) $(1,1)$
(3) If $\overrightarrow{\mathrm{F}_{1}}=4 \overrightarrow{\mathrm{i}}, \overrightarrow{\mathrm{F}_{2}}=8 \overrightarrow{\mathrm{i}}-5 \vec{j}$, then $\|\overrightarrow{\mathrm{R}}\|=\cdots \ldots \ldots$ force unit.
(a) 12
(b) 5
(c) 13
(d) $\sqrt{73}$
(4) If $\overrightarrow{F_{1}}=3 \vec{i}+2 \vec{j}, \overrightarrow{F_{2}}=a \vec{i}+7 \vec{j}, \overrightarrow{F_{3}}=-12 \vec{i}+b \vec{j}$ are three coplanar forces meeting at a point and the resultant $\stackrel{\rightharpoonup}{\mathrm{R}}=\left(6 \sqrt{2}, \frac{3}{4} \pi\right)$, then $\mathrm{a}-\mathrm{b}=$ $\qquad$
(a) -3
(b) 3
(c) zero
(d) 6
(5) Three coplanar forces $\overrightarrow{F_{1}}=6 \vec{i}+7 \vec{j}, \overrightarrow{F_{2}}=a \vec{i}-9 \vec{j}, \overrightarrow{F_{3}}=5 \vec{i}+b \vec{j}$ act at a particle and they are in equilibrium, then $\mathrm{a}+2 \mathrm{~b}=$
(a) -9
(b) 5
(c) 7
(d) -7
(6) If $\overrightarrow{\mathrm{F}}_{1}, \overrightarrow{\mathrm{~F}}_{2}$ and $\overrightarrow{\mathrm{F}}_{3}$ are three coplanar equilibrium forces meeting at a point, and $\vec{F}_{1}=2 \vec{i}-3 \vec{j}, \overrightarrow{F_{2}}=3 \vec{i}+5 \vec{j}$, then $\overrightarrow{F_{3}}=$ $\qquad$
(a) $-5 \overrightarrow{\mathrm{i}}-2 \overrightarrow{\mathrm{j}}$
(b) $-5 \vec{i}+2 \vec{j}$
(c) $5 \vec{i}+2 \overrightarrow{\mathrm{j}}$
(d) $5 \overrightarrow{\mathrm{i}}-2 \overrightarrow{\mathrm{j}}$
( 7 ) If the resultant of the forces in the given figure acts in direction of y -axis, then $\mathrm{F}=$ $\qquad$ force unit.
(a) 2
(b) 6
(c) 8
(d) 14

( 8 ) The resultant of the forces in the opposite figure acts in direction
of $\qquad$ ...
(a) $\overrightarrow{C D}$
(b) $\overrightarrow{\mathrm{CE}}$
(c) $\overrightarrow{\mathrm{CF}}$
(d) $\overrightarrow{\mathrm{CA}}$

(9) In the opposite figure:

The magnitude of four coplanar forces are $1,2,4 \sqrt{3}, 3 \sqrt{3}$ newton act at point O in the direction of $\overrightarrow{\mathrm{OX}}, \overrightarrow{\mathrm{OA}}, \overrightarrow{\mathrm{OB}}$ and $\overrightarrow{\mathrm{OY}}$ $, \mathrm{m}(\angle \mathrm{AOC})=60^{\circ}, \mathrm{m}(\angle \mathrm{BOD})=30^{\circ}$, then the magnitude and the direction of the resultant of the forces is $\qquad$

(a) $\left(4,180^{\circ}\right)$
(b) $\left(4,0^{\circ}\right)$
(c) $\left(3,0^{\circ}\right)$
(d) $\left(5,90^{\circ}\right)$
(10) In the opposite figure :

ABCD is a square, the forces of magnitudes
$5,8,4 \sqrt{2}$ newton act on $\overrightarrow{\mathrm{AB}}, \overrightarrow{\mathrm{AD}}$ and $\overrightarrow{\mathrm{AC}}$ respectively , then the polar form of the resultant is $\qquad$
(a) $\left(5,54^{\circ}\right)$
(b) $\left(15,60^{\circ}\right)$
(c) $\left(15,53^{\circ} \stackrel{\circ}{8}\right)$
(d) $\left(13,90^{\circ}\right)$


(a) South.
(b) East.
(c) West.
(b) East.
(d) North.
(11) In the opposite figure :

The direction of the resultant of the forces is $\qquad$ ...

## (12) In the opposite figure :

The magnitude of the resultant of the forces $(\mathrm{R})=$ $\qquad$ newton.
(a) 20
(b) $10 \sqrt{2}$
(c) 10
(d) zero

(13) In the opposite figure :

Five equal forces each of magnitude 10 newton act at one vertex of a regular hexagon and in direction of the other vertices of the hexagon, then the magnitude of the resultant of these forces $=$ $\qquad$ newton.
(a) 50
(b) 20
(c) $30 \sqrt{3}$
(d) $20+10 \sqrt{3}$

(14) In the opposite figure :

ABCDEF is a regular hexagon, the forces of magnitudes $15,5 \sqrt{3}, 5 \sqrt{3}, 15$ newton act on $\overrightarrow{\mathrm{AB}}, \overrightarrow{\mathrm{CA}}, \overrightarrow{\mathrm{EA}}, \overrightarrow{\mathrm{AF}}$ respectively, then the magnitude of their resultant $=$ $\qquad$ newton.
(a) 5
(b) 10
(c) 25
(d) zero

(15) In the opposite figure :

ABCDEF is a regular hexagon, forces of magnitudes $2,4 \sqrt{3}, 8,2 \sqrt{3}$ and 4 kg .wt. act at point A in directions $\overrightarrow{\mathrm{AB}}, \overrightarrow{\mathrm{AC}}, \overrightarrow{\mathrm{AD}}, \overrightarrow{\mathrm{AE}}$ and $\overrightarrow{\mathrm{AF}}$ respectively.
First : The magnitude of their resultant $=$ $\qquad$
(a) $14+6 \sqrt{3}$
(b) 20
(c) $20 \sqrt{3}$
(d) $20+\sqrt{3}$


Second : The direction of the resultant inclined by an angle of measure $\qquad$ with $\overrightarrow{\mathrm{AB}}$
(a) $30^{\circ}$
(b) $45^{\circ}$
(c) $60^{\circ}$
(d) $90^{\circ}$
(16) If the resultant of the forces represented in the opposite figure acts in $X$-axis , then $\mathrm{F}=$ $\qquad$ newton.
(a) 10
(b) 14
(c) 18
(d) 6

(17) The opposite figure represents some of forces meeting at a point, then the magnitude of the resultant of these forces $=$ $\qquad$ newton.
(a) $15 \sqrt{2}$
(b) 5
(c) $5 \sqrt{2}-5$
(d) zero


5 newton
(18) Three coplanar forces meeting at a point, their magnitudes are $40,30,40$ newton , the first is in direction $60^{\circ}$ West of North, the second is towards West and the third in the direction $30^{\circ}$ North of East, then the magnitude of their resultant equal newton.
(a) 30
(b) 110
(c) 60
(d) 50
(19) In the opposite figure :
$A B C D$ is a rectangle $A B=4 \mathrm{~cm} ., B C=3 \mathrm{~cm}$. forces $4 \mathrm{~N}, 10,6 \mathrm{~N}$ acts along $\overrightarrow{\mathrm{AB}}, \overrightarrow{\mathrm{AC}}, \overrightarrow{\mathrm{AD}}$ respectively. The resultant of these forces makes with $\overrightarrow{\mathrm{AB}}$ an angle of measure $\qquad$

(a) $45^{\circ}$
(b) $60^{\circ}$
(c) $30^{\circ}$
(d) $\sin ^{-1}\left(\frac{3}{5}\right)$
(20) $A B C D$ is a right trapezium at $A$ and $D$, in which $A D=C D=4 \mathrm{~cm}$., $A B=7 \mathrm{~cm}$. , $M \in \overline{\mathrm{AB}}$ where $\mathrm{AM}=4 \mathrm{~cm}$, a set of forces their magnitudes $25, F$ and $15 \sqrt{2}$ gm.wt. act at $\overrightarrow{\mathrm{CB}}, \overrightarrow{\mathrm{CM}}$ and $\overrightarrow{\mathrm{CA}}$ respectively and the norm of the resultant of these forces equals 45 gm.wt., then the value of $\mathrm{F}=$ $\qquad$ gm.wt.
(a) 10
(b) 50
(c) 20
(d) 30
(21) The forces of magnitudes $F, 12,8 \sqrt{2}, 10 \sqrt{2}, \mathrm{k}$ newton act on a particle in the directions of East, North, Western North, Western South and South respectively. If the magnitude of the resultant $=4$ newton due to North, then $\mathrm{F}-\mathrm{K}=$ newton
(a) 24
(b) 27
(c) 12
(d) 6
$\therefore$ (22) In the opposite figure :
The forces of magnitude $\mathrm{F}, 5, \mathrm{~K}$ and $6 \sqrt{10} \mathrm{~N}$ act in the rectangle ABCD in the directions $\overrightarrow{\mathrm{CB}}, \overrightarrow{\mathrm{CA}}, \overrightarrow{\mathrm{CD}}, \overrightarrow{\mathrm{HC}}$

Such that : $\mathrm{AB}=6 \mathrm{~cm}$., $\mathrm{BC}=8 \mathrm{~cm} ., \mathrm{AH}=6 \mathrm{~cm}$.
If these forces are in equilibrium, then $K=$ $\qquad$

(a) 12
(b) 15
(c) 18
(d) 20
(23) The coplanar forces of magnitudes $5,4, \mathrm{~F}, 3, \mathrm{k}, 7 \mathrm{~kg} . \mathrm{wt}$. act at a particle and the measure of the angle between each two consecutive forces is $60^{\circ}$, if the system is in equilibrium, then $\mathrm{F}+2 \mathrm{~K}=$ $\qquad$ kg.wt.
(a) 21
(b) 6
(c) 9
(d) 15
$\therefore$ (24) The opposite figure represents a set of forces meeting at a point (O)
Mohamed took ( O ) as an origin of coordinate system and the positive direction of $X$-axis in direction of $\overrightarrow{\mathrm{F}}_{1}$ The magnitude of the resultant was $\mathrm{R}_{1}$ and made angle of measure $\left(\theta_{1}\right)$ with the positive direction of $X$-axis and Ebrahim took $(\mathrm{O})$ as an origin of coordinate system and the positive direction of $X$-axis in direction of $\overrightarrow{\mathrm{F}_{2}}$, the magnitude
 of the resultant was $R_{2}$ and made an angle of measure $\left(\theta_{2}\right)$ with the positive direction of $X$-axis, then $\qquad$
(a) $\mathrm{R}_{1}=\mathrm{R}_{2}, \theta_{1}=\theta_{2}$
(b) $\mathrm{R}_{1}=\mathrm{R}_{2}, \theta_{1} \neq \theta_{2}$
(c) $R_{1} \neq R_{2}, \theta_{1}=\theta_{2}$
(d) $R_{1} \neq R_{2}, \theta_{1} \neq \theta_{2}$

## Second Essay questions

1 Find the resultant (magnitude and direction) of the set of forces in each of the following figures (where each force magnitude is in newton) :
(1)

(2)


2 Three coplanar forces of magnitudes $1,2, \sqrt{3}$ newton act at M , their directions are $\overrightarrow{\mathrm{MA}}$ , $\overrightarrow{\mathrm{MB}}$ and $\overrightarrow{\mathrm{MC}}$ respectively where $\mathrm{m}(\angle \mathrm{AMB})=60^{\circ}, \mathrm{m}(\angle \mathrm{BMC})=30^{\circ}$
, $\mathrm{m}(\angle \mathrm{AMC})=90^{\circ}$, find the resultant.
«4 newton, in direction of $\overrightarrow{\mathrm{MB}}$ »
3 The forces $8,4 \sqrt{3}, 6 \sqrt{3}$ and 14 newton act at a point, the measure of the angle between the first force and the second force is $30^{\circ}$, between the second and the third is $120^{\circ}$ and between the third and the fourth is $90^{\circ}$ taken in the same cyclic order. Find the magnitude and direction of the resultant of these forces.
« 4 newton, in direction of $4^{\text {ti }}$ force »
4 The coplanar forces of magnitudes $2,3 \sqrt{2}, 2 \sqrt{3}$ and $\sqrt{3}$ newton act at a point. If the measures between the first force and the second force is $45^{\circ}$, the measure between the second and the third is $105^{\circ}$ and the measure between the third and the fourth is $120^{\circ}$ taken in the same cyclic order, find the resultant of these forces.
5. Five coplanar forces meeting at a point, their magnitudes are $9,6,4 \sqrt{2}, 5 \sqrt{2}$ and 5 newton act due to East, North , Western North, Western South and in the direction of South respectively. Prove that the set of forces are in equilibrium.

6 Three coplanar forces of magnitudes 60,88 and 60 gm.wt. act at a point, the $1^{\text {st }}$ is towards North, the second is in the direction $30^{\circ}$ South of West and the $3^{\text {rd }}$ in the direction $30^{\circ}$ South of East.
Find the magnitude of the resultant of these forces and its direction.
«28 gm.wt. , $30^{\circ}$ South of West»
1 CD Four coplanar forces act on a particle the first of magnitude 4 newton acts in the Eastern direction, the second of magnitude 2 newton, acts in direction $60^{\circ}$ North of the East, the third of magnitude 5 newton, acts in direction $60^{\circ}$ North of the West and the fourth of magnitude $3 \sqrt{3}$ newton acts in direction $60^{\circ}$ West of the South. Find the magnitude and direction of their resultant.

8 The forces of magnitudes $2 \mathrm{~F}, 3 \mathrm{~F}$ and 4 F newton act on a particle in the directions parallel to the sides of an equilateral triangle in the same cyclic order.
Find the magnitude and the direction of the resultant of these forces.
« $\sqrt{3} \mathrm{~F}$ newton, perpendicular to the force 3 F »
9 DD ABC is an equilateral triangle. M is the point of intersection of its medians.
the forces of magnitude 15,20 and 25 newton act on a particle at the point M in the directions of $\overrightarrow{\mathrm{MC}}, \overrightarrow{\mathrm{MB}}, \overrightarrow{\mathrm{MA}}$
Find the magnitude and the direction of the resultant of these forces.
(10) $\triangle \mathrm{ABC}$ is an isosceles triangle where $\mathrm{m}(\angle \mathrm{BAC})=120^{\circ}$, the forces of magnitudes $4,6 \sqrt{3}, 4$ newton act at A in the directions $\overrightarrow{\mathrm{AB}}, \overrightarrow{\mathrm{CB}}, \overrightarrow{\mathrm{CA}}$ respectively. Find the magnitude and the direction of the resultant of these forces.
« $10 \sqrt{3}$ newton in the direction of $\overrightarrow{\mathrm{CB}}$ *
11 Four coplanar forces of magnitude $2,1,4$ and $3 \sqrt{3} \mathrm{~N}$. act at a point A in directions of $\overrightarrow{\mathrm{BC}}, \overrightarrow{\mathrm{BA}}, \overrightarrow{\mathrm{CA}}$ and $\overrightarrow{\mathrm{AD}}$ where ABC is an equilateral triangle and D is the midpoint of $\overrightarrow{\mathrm{BC}}$ Find the magnitude and direction of their resultant. «1 newton in the direction of $\overrightarrow{\mathrm{AC}}$.

12 ABCD is a rectangle where $\mathrm{AB}=4 \mathrm{~cm}$. , $\mathrm{BC}=3 \mathrm{~cm}$. the forces of magnitudes 2,5 and 3 kg .wt. act at the point A in the directions $\overrightarrow{\mathrm{AB}}, \overrightarrow{\mathrm{AC}}$ and $\overrightarrow{\mathrm{AD}}$ respectively.
Find the resultant of these forces and the measure of its angle of inclination on $\overrightarrow{\mathrm{AB}}$
(18) ABCD is a rectangle in which $\mathrm{AB}=8 \mathrm{~cm}$. , $\mathrm{BC}=6 \mathrm{~cm}$., $\mathrm{E} \in \overline{\mathrm{CD}}$ where $\mathrm{ED}=6 \mathrm{~cm}$., a set of forces their magnitudes $12,40,26 \sqrt{2}$ and 4 newton act at $\overrightarrow{A B}, \overrightarrow{\mathrm{CA}}, \overrightarrow{\mathrm{AE}}$ and $\overrightarrow{\mathrm{AD}}$ respectively.
Find the magnitude and the direction of the resultant of these forces.

14 ABCD is a rectangle in which : $\mathrm{AB}=21 \mathrm{~cm}$. , $\mathrm{BC}=9 \mathrm{~cm}$. The point $\mathrm{O} \in \overline{\mathrm{AB}}$ where $\mathrm{AO}=9 \mathrm{~cm}$. four forces of magnitudes $4,10,6$ and $12 \sqrt{2} \mathrm{~kg}$.wt. act at the point O in the directions $\overrightarrow{\mathrm{OB}}, \overrightarrow{\mathrm{OC}}, \overrightarrow{\mathrm{BC}}$ and $\overrightarrow{\mathrm{OD}}$ respectively.
Find the magnitude of the resultant of these forces and prove that it is parallel to $\overrightarrow{\mathrm{BC}}$

1-7 ABCDEF is a regular hexagon, the forces of magnitudes $8,6 \sqrt{3}, 5,4 \sqrt{3}$ newton act on $\overrightarrow{\mathrm{AB}}, \overrightarrow{\mathrm{AC}}, \overrightarrow{\mathrm{AD}}$ and $\overrightarrow{\mathrm{AE}}$ respectively. Find the magnitude and the direction of their resultant.
« $\sqrt{651}$ newton, $40^{\circ} 9$ with $\overrightarrow{\mathrm{AB}}$.
10 ABCDHE is a regular hexagon. Forces of magnitudes $2,4 \sqrt{3}, 8,2 \sqrt{3}$ and 4 kg .wt. act at point A in directions $\overrightarrow{\mathrm{AB}}, \overrightarrow{\mathrm{AC}}, \overrightarrow{\mathrm{AD}}, \overrightarrow{\mathrm{AH}}, \overrightarrow{\mathrm{AE}}$ respectively.
Find the magnitude and the direction of their resultant.
« $20 \mathrm{~kg} . \mathrm{wt}$., $60^{\circ}$ with $\overrightarrow{\mathrm{AB}}$.
17 ABCDEF is a regular hexagon. M is the point of intersection of its diagonals. the forces of magnitudes $4,1,4,5,2$ and $3 \mathrm{gm} . \mathrm{wt}$. act at M in the directions of $\overrightarrow{\mathrm{MA}}, \overrightarrow{\mathrm{MB}}, \overrightarrow{\mathrm{MC}}, \overrightarrow{\mathrm{MD}}, \overrightarrow{\mathrm{ME}}$ and $\overrightarrow{\mathrm{MF}}$
Find the resultant of these forces and prove that it is in the direction of $\overrightarrow{M D}$

18 ABC is a right-angled triangle at B where $\mathrm{AB}=80 \mathrm{~cm}$. , $\mathrm{BC}=60 \mathrm{~cm}$. , $\mathrm{D} \in \overline{\mathrm{AC}}$ where BD = DC
The four forces of magnitudes $8,12,15$ and 10 newton act at the point B in the directions $\overrightarrow{\mathrm{AB}}, \overrightarrow{\mathrm{BC}}, \overrightarrow{\mathrm{CA}}$ and $\overrightarrow{\mathrm{BD}}$ respectively.
Find the resultant of these forces and prove that it acts in $\overrightarrow{\mathrm{BD}}$
$19 . \mathrm{ABCD}$ is a square of side length is $12 \mathrm{~cm} . \mathrm{H} \in \overline{\mathrm{BC}}$ where $\mathrm{BH}=5 \mathrm{~cm}$. forces of magnitudes $2,13,4 \sqrt{2}, 9$ gm.wt. act in directions of $\overrightarrow{\mathrm{AB}}, \overrightarrow{\mathrm{AH}}, \overrightarrow{\mathrm{CA}}$ and $\overrightarrow{\mathrm{AD}}$ respectively.
Find the magnitude of the resultant of these forces.
20 ABCD is a square of side length 6 cm . The point E is the midpoint of $\overline{\mathrm{BC}}$ and F is the midpoint of $\overline{\mathrm{DC}}$, the five forces of magnitudes $2,12 \sqrt{5}, 6 \sqrt{2}, 4 \sqrt{5}$ and 4 kg .wt. act at the point A in the directions of $\overrightarrow{\mathrm{AB}}, \overrightarrow{\mathrm{AE}}, \overrightarrow{\mathrm{CA}}, \overrightarrow{\mathrm{AF}}$ and $\overrightarrow{\mathrm{AD}}$ respectively.
Find the magnitude and the direction of the resultant of these forces. « $30 \mathrm{~kg} . \mathrm{wt}, ~ 36^{\circ} 5212$,
21 ABCD is a square, $\mathrm{E} \in \overline{\mathrm{AD}}$, four forces of magnitudes $4,4 \sqrt{3}, 10 \sqrt{2}, \mathrm{~F}$ kg.wt. act at point $B$ in the directions $\overrightarrow{\mathrm{BA}}, \overrightarrow{\mathrm{BE}}, \overrightarrow{\mathrm{DB}}, \overrightarrow{\mathrm{BC}}$, if these forces are in equilibrium, find $m(\angle \mathrm{ABE})$ and the value of $F$

22 The coplanar forces of magnitudes $5,4, \mathrm{~F}, 3, \mathrm{~K}$ and 7 kg .wt. act at a particle and the measure of the angle between each two consecutive forces is $60^{\circ}$ Find the magnitude of F and K that makes the system in equilibrium.

23 The forces of magnitudes $F, 6,4 \sqrt{2}, 5 \sqrt{2}$, K newton act on a particle in the directions of East, North , Western North , Western South and South respectively. Find the values of F and K if the magnitude of the resultant $=2$ newton due to North.

24 Forces of magnitudes $F, 4 \sqrt{3}, 12 \sqrt{3}, 36$ gm.wt. act at a particle. The last three forces are in the directions of North, $60^{\circ}$ West of North, $60^{\circ}$ South of East respectively. If the resultant of these four forces $=8 \mathrm{gm} . \mathrm{wt}$. in magnitude in the direction of East.
Determine the value of F and its direction.
« 16 gm.wt., $60^{\circ}$ North of East »
2 The forces of magnitudes $\mathrm{F}, 8, \mathrm{~K}, 5,8 \sqrt{3}$ newton act at a point in the directions of : East, $30^{\circ}$ East of North, North, West and South respectively.
Find the values of F and K if the resultant is 4 newton in magnitude in the direction of $60^{\circ}$ North of East.
$26^{6} \mathrm{ABCD}$ is a right trapezium at A and D , in which $\mathrm{AD}=\mathrm{CD}=40 \mathrm{~cm} ., \mathrm{AB}=70 \mathrm{~cm}$., $\mathrm{M} \in \overline{\mathrm{AB}}$ where $\mathrm{AM}=40 \mathrm{~cm}$., a set of forces their magnitudes $25, F, 10 \sqrt{2}$ and 35 gm .wt. act at $\overrightarrow{\mathrm{CB}}, \overrightarrow{\mathrm{CM}}, \overrightarrow{\mathrm{CA}}$ and $\overrightarrow{\mathrm{CD}}$ respectively and the norm of the resultant of these forces equals 50 gm.wt. Find $F$

21 In each of the following figures find the magnitudes of F and K in newton that makes the system in equilibrium :
(1)

(2)

(3)


28 Coplanar forces of magnitudes $F, 3 \sqrt{2}, 2 \sqrt{3}$ and $\sqrt{3}$ newton act on a particle.
The first force acts in the east direction. The angle between the first and the second force is of measure $45^{\circ}$, the angle between the second and the third force is of measure $105^{\circ}$ , the angle between the third and the fourth force is of measure $120^{\circ}$. If the magnitude of their resultant is $3 \sqrt{2}$ newton, then find the value of $F$ and measure of the angle between the resultant and the first force.
29) ABCDEF is a regular hexagon.

Forces of magnitudes $4,2 \sqrt{3}, F, 2 \sqrt{3}$ and K kg.wt. act in the directions of $\overrightarrow{\mathrm{AB}}, \overrightarrow{\mathrm{AC}}$, $\overrightarrow{\mathrm{AD}}, \overrightarrow{\mathrm{AE}}$ and $\overrightarrow{\mathrm{AF}}$ respectively.
If the resultant of these forces is of magnitude 20 kg .wt. in the direction of $\overrightarrow{\mathrm{AD}}$ Find the values of $\mathrm{F}, \mathrm{K}$

## 30 In the opposite figure :

Four coplanar forces act at the point (O) in the directions shown in the figure where $\sin \theta=\frac{4}{5}$ and the resultant of these forces is $8 \sqrt{2} \mathrm{~N}$. and makes an angle of measure $135^{\circ}$ with $\overrightarrow{\mathrm{OX}}$ , then find the values of $\mathrm{F}, \mathrm{K}$


31 If $\overrightarrow{F_{1}}=5 \vec{i}+3 \vec{j}, \overrightarrow{F_{2}}=a \vec{i}+6 \vec{j}, \overrightarrow{F_{3}}=-14 \vec{i}+b \vec{j}$ are three coplanar forces meeting at a point and their resultant is $\overrightarrow{\mathrm{R}}=\left(10 \sqrt{2}, 135^{\circ}\right)$, then find the values of a and b «a=-1, $\mathrm{b}=1$ »

## FIRST

## Monthly Tests of October

## Test

 11 Choose the correct answer from the given ones :
(1) Two forces of magnitudes 8 and $16 \mathrm{~kg} . \mathrm{wt}$. and the measure of their included angle is $120^{\circ}$ If these two forces act at a body, then the direction of motion of the body makes an angle of measure $\qquad$ with the smaller force.
(a) $30^{\circ}$
(b) $90^{\circ}$
(c) $60^{\circ}$
(d) $45^{\circ}$
(2) Two forces of equal magnitude and intersecting at a point. The measure of the angle between the two forces is $120^{\circ}$ and the magnitude of each is 6 N ., then the magnitude of their resultant $=$ $\qquad$
(a) 12
(b) $6 \sqrt{3}$
(c) 6
(d) $12 \sqrt{3}$
(3) F N. and K N. are the magnitudes of two forces where F $>\mathrm{K}$ If the smallest and the greatest value of their resultant are 5,9 newton respectively , then $5 \mathrm{~F}-2 \mathrm{~K}=$ $\qquad$ N .
(a) 53
(b) 31
(c) 49
(d) 4
(4) A body of weight 20 N . is placed on a smooth inclined plane makes an angle of measure $30^{\circ}$ with the horizontal, then the component of the weight in direction perpendicular to the plane $=$ $\qquad$ N.
(a) 10
(b) 20
(c) $10 \sqrt{2}$
(d) $10 \sqrt{3}$
(5) Forces of magnitudes $8,4 \sqrt{3}, 6 \sqrt{3}, 14$ newton act at a point. The measure of the angle between the first and second force is $30^{\circ}$ and between the second and third is $120^{\circ}$ and between the third and fourth is $90^{\circ}$ in one cyclic order, then the magnitude of their resultant $=$ $\qquad$
(a) 4
(b) 6
(c) 8
(d) 7
(6) Two forces of magnitudes $3, F$ newton and measure of the angle between them is $\frac{2 \pi}{3}$ if their resultant is perpendicular to the first force, then $\mathrm{F}=$ $\qquad$ newton.
(a) 1.5
(b) 3
(c) $3 \sqrt{2}$
(d) 6

2 Answer the following questions:
(1) A force of magnitude 18 newton acts in south direction. Find its two components in directions of $60^{\circ}$ East of South and $30^{\circ}$ West of South.
(2 marks)
(2) Three coplanar forces of magnitudes $1,2, \sqrt{3}$ newton act at $M$, their directions are $\overrightarrow{\mathrm{MA}}, \overrightarrow{\mathrm{MB}}$ and $\overrightarrow{\mathrm{MC}}$ respectively where $\mathrm{m}(\angle \mathrm{AMB})=60^{\circ}, \mathrm{m}(\angle \mathrm{BMC})=30^{\circ}$ , $\mathrm{m}(\angle \mathrm{AMC})=90^{\circ}$ Find the resultant.

## Test

2

## 1 Choose the correct answer from the given ones :

(6 marks)
(1) The resultant of two forces 6,8 newton is 10 N ., then the measure of the angle between their directions $=$ $\qquad$ .
(a) 60
(b) 90
(c) 120
(d) 150
(2) Two forces intersecting at a point, their magnitudes 7 and F newton and their resultant bisects the angle between them, then $(\mathrm{F}-1)=$ N .
(a) 8
(b) 7
(c) 6
(d) 5
(3) In the opposite figure:

The force $\vec{R}$ is resolved into two components $\vec{F}_{1}$ and $\overrightarrow{F_{2}}$ , then $\mathrm{F}_{1}=$ $\qquad$ newton.
(a) $12 \cos 75^{\circ}$
(b) $12 \cos 45^{\circ}$
(c) $6 \csc 45^{\circ}$
(d) $6 \csc 75^{\circ}$

( 4 ) In the opposite figure :
If the resultant of the shown forces acts in direction of $y$-axis, then $F=$ $\qquad$ N.
(a) 2
(b) 6
(c) 8
(d) 14

( 5 ) The magnitudes of two forces are 5 and 10 newton and their resultant is perpendicular on the smaller force. If the measure of angle between the two forces is $\alpha$ and their resultant is $\mathbb{R}$, then $\qquad$
(a) $\alpha=60^{\circ}, \mathbb{R}=10 \sqrt{3} \mathrm{~N}$.
(b) $\alpha=120^{\circ}, \mathbb{R}=10 \sqrt{3} \mathrm{~N}$.
(c) $\alpha=60^{\circ}, \mathbb{R}=5 \sqrt{3} \mathrm{~N}$.
(d) $\alpha=120^{\circ}, \mathbb{R}=5 \sqrt{3} \mathrm{~N}$.
( 6 ) In the opposite figure :
A body of weihgt 260 gm.wt. and $\tan \theta=\frac{5}{12}, W_{1}, W_{2}$ are magnitudes of the two components in direction of the inclined plane downward and perpendicular to the plane, then

(a) $\mathrm{W}_{1}=120$ gm.wt., $\mathrm{W}_{2}=50$ gm.wt.
(c) $\mathrm{W}_{1}-\mathrm{W}_{2}=70$ gm.wt.
(d) $\mathrm{W}_{1}+\mathrm{W}_{2}=340$ gm.wt.

## Monthly tests

2 Answer the following questions:
(1) In the opposite figure :

If the force of magnitude 40 N . is resolved into two components $\vec{F}_{1}$ and $\vec{F}_{2}$ as shown in the figure.

(2 marks)
( 2 ) The magnitudes of three forces are $10,20,30$ newton acting at one point. The first acts due east, the second makes an angle of measure $30^{\circ}$ west of the north and the third makes an angle of measure $60^{\circ}$ south of the west. Find the magnitude and the direction of their resultant.
(2 marks)

## Answers of October tests

## Answers of Test 1

(1) b
(2) c
(3) b
(4) d
(5) a
(6) d
(1) $\because$ The two components are perpendicular
$\therefore \mathrm{F}_{1}=18 \cos 60^{\circ}$
$=9$ newton
, $\mathrm{F}_{2}=18 \sin 60^{\circ}=9 \sqrt{3}$ newton
(2)


Consider $\overrightarrow{\mathrm{OX}}$ is the direction of the first force.
$X=1 \times \cos 0^{\circ}+2 \cos 60^{\circ}+\sqrt{3} \cos 90^{\circ}$
$=1 \times 1+2 \times \frac{1}{2}+\sqrt{3} \times 0=2$
$\mathrm{Y}=1 \times \sin 0^{\circ}+2 \times \sin 60^{\circ}+\sqrt{3} \sin 90^{\circ}$
$=1 \times 0+2 \times \frac{\sqrt{3}}{2}+\sqrt{3} \times 1=2 \sqrt{3}$
$\therefore \overrightarrow{\mathrm{R}}=2 \overrightarrow{\mathrm{i}}+2 \sqrt{3} \overrightarrow{\mathrm{j}}, \mathrm{R}=\sqrt{(2)^{2}+(2 \sqrt{3})^{2}}$

$$
=4 \text { newton }
$$

, $\tan \theta=\frac{2 \sqrt{3}}{2}=\sqrt{3}$
, $\because \mathrm{X}>0, \quad \mathrm{Y}>0$
$\therefore$ The magnitude of $\vec{R}=4$ newton and its direction is $\overrightarrow{\mathrm{MB}}$

## Answers of Test 2

(1) b
(2) c
(3) d
(4) b
(5) d
(6) d

2
(1) From the figure
$\sin \theta=0.8, \cos \theta=0.6$
$F_{1}=\frac{40 \sin \left(90^{\circ}-\theta\right)}{\sin \left(180^{\circ}-\theta\right)}$


$$
=\frac{40 \cos \theta}{\sin \theta}=\frac{40 \times 0.6}{0.8}=30 \mathrm{~N} .
$$

$$
, \mathrm{F}_{2}=\frac{40 \sin 90^{\circ}}{\sin \left(180^{\circ}-\theta\right)}=\frac{40 \times 1}{\sin \theta}=\frac{40}{0.8}=50 \mathrm{~N} .
$$

(2) $\mathrm{X}=10 \cos 0^{\circ}+20 \cos 120^{\circ}$
$+30 \cos 240^{\circ}=-15$
$Y=10 \sin 0^{\circ}+20 \sin 120^{\circ}$
$+30 \sin 240^{\circ}=-5 \sqrt{3}$ Wen
$\therefore \stackrel{\rightharpoonup}{\mathrm{R}}=-15 \stackrel{\rightharpoonup}{\mathrm{i}}-5 \sqrt{3} \overrightarrow{\mathrm{j}}$
$\therefore \mathrm{R}=\sqrt{225+75}$


$$
=10 \sqrt{3} \mathrm{~N}
$$

$\tan \theta=\frac{y}{x}=\frac{-5 \sqrt{3}}{-15}=\frac{1}{\sqrt{3}}$
, $\because x<0, \mathrm{y}<0$
$\therefore \theta=180^{\circ}+30^{\circ}=210^{\circ}$
i.e. In direction $30^{\circ}$ South of West.

## Complete the following:

The effect of a force on a body is determined by the following:
The vector of the resultant of the two forces $\mathrm{F}_{1}, \mathrm{~F}_{2}$ is equal to : $\qquad$
The maximum value of the resultant of two forces of magnitudes 4,6 Newton meeting at a point equals $\qquad$
The minimum value of the resultant of two forces of magnitudes 5, 9 Newton meeting at a point equals $\qquad$
2,3 Newton are twe forces, if the angle between them is 60 then the magnitude of their resultant equals

## Choose the correct answer from those given:

The magnitude of the resultant of the two forces of magnitudes 3,5 newton and the measure of the angle between them is 60 equals
A 2 N
B 6 N
a 7 N
D 8 N

Two forces of magnitudes 3, 4 N act on a particle and the magnitude of their resultant is 5 N , then the measure of the angle between them equals
A 30
B 45
a 60
D 90

Two equal forces, the magnitude of each of them is 6 N , the magnitude of their resultant is 6 N , then the angle between them equals:
(A) 30
B 60
a 120
D 150

Two forces of magnitudes 3, F Newton and the measure of the angle between them is 120 . If their resultant is perpendicular to the first force, so the value of F in Newton is
(A) 1.5
B 3
(a) $3 \sqrt{3}$
D 6

If the two forces $6,8 \mathrm{~N}$ are perpendicular then the sine of the angle of inclination of their resultant with the first force equals:
(A) $\frac{3}{5}$
(B) $\frac{4}{5}$
(a) $\frac{3}{4}$
(D) $\frac{4}{3}$

## Answer the following questions:

Two forces of magnitudes 5,10 Newton act on a particle and the measure of the angle between them is 120 . Find the magnitude of their resultant and the measure of the angle made by the resultant with the first force.

Two forces of magnitudes $3,3 \sqrt{2} \mathrm{~kg}$.wt act on a particle and the measure of the angle between them is 45 . Find the magnitude and the direction of their resultant.

Two forces of magnitudes $15,8 \mathrm{~kg}$.wt act on a particle. If their resultant equals $13 \mathrm{~kg} . \mathrm{wt}$, find the angle between the two forces.

Two forces of magnitudes 8 , F Newton act on a particle and measure of the angle between them is 120 . If their resultant is $\mathrm{F} \sqrt{3} \mathrm{~N}$, find the magnitude of F .

Two forces of magnitudes 4 , F Newton act on a particle and the measure of angle between them is 135 , If the direction of their resultant is inclined by an angle of measure 45 on F . Find f

## Forces resolution

## Complete the following:

A force of magnitude 6 Newton acts in direction of North. It is resolved into two perpendicular components, so its component in direction of the East equals $\qquad$ Newton.

A force of magnitude $4 \sqrt{2}$ newton acts in direction of East. It is resolved into two Perpendicular components, so its component in the direction of Northern East equals $\qquad$ … N Newton.
\# If the force $R$ is resolved into two components $F_{1}, F_{2}$ which make with the force R two angles of measures 30 , 45 from different directions of its line of action, $\|\mathrm{R}\|=12$ newton,

So: $\mathrm{F}_{1}=$ $\qquad$ Newton, $\mathrm{F}_{2}=$ $\qquad$ Newton.
\# If the force $R$ is resolved into two components $F_{1}$ , $\mathrm{F}_{2}$ which make with the force R two angles of measure 45,90 from different directions of its line of action and $\|R\|=18$ Newton, So: $\mathrm{F}_{1}=\ldots . . . .$. Newton, $\mathrm{F}_{2}=$ $\qquad$ Newton

If the force $F$ is resolved into two perpendicular components $F_{1}, F_{2}$ and the force vecor $F$ bisects the between the directions of $\mathrm{F}_{1}, \mathrm{~F}_{2}$ and $\|\mathrm{F}\|=6 \sqrt{2} \mathrm{~kg}$. wt
so: $\quad\left\|\mathrm{F}_{1}\right\|=$ $\qquad$ kg wt ,
|| $\mathrm{F}_{2} \|=$ $\qquad$ kg wt.

Force of magnitude $12 \sqrt{2}$ newton acts in direction 30 North of the west.

3/4 Magnitude of the component of the force in the western direction $=$. $\qquad$ Newton.
$3 / 4$ Magnitude of the component of the force in the northern direction $=$ $\qquad$ Newton.

A force of magnitude $600 \mathrm{gm} . \mathrm{wt}$ acts on a particle. Find its two components in two directions making with the force two angles of measures 30,45 .

A force of magnitude 120 newton acts in direction of the Northeast. Find its two components in the direction of East and in the direction of North.

A rigid body of weight 42 newton is placed on a plane inclined to the horizontal with a angle of measure 60. Find the two components of the weight of the body in the direction of the line of the greatest slope and the direction normal t

## The resultant of coplanar forces meeting at a point

## Exercise

## Complete the following:

If the forces $\mathrm{F}_{1}=2 i, \mathrm{~F}_{2}=i-2 j, \quad \mathrm{~F}_{3}=6 j$ then:
the magnitude of the resultant of the forces $=$ $\qquad$ and its direction $=$ $\qquad$

If the forces $\mathrm{F}_{1}=2 i-2 j, \mathrm{~F}_{2}=4 i-8 j, \quad \mathrm{R}=2 \mathrm{a} i-3 \mathrm{~b} j$ then: $\mathrm{a}=$ $\qquad$ $\mathrm{b}=$ $\qquad$

|  | $\begin{array}{lllll}3 & i-2 j & , \mathrm{~F}_{2}=\mathrm{a} i-j \\ = & \mathrm{b}=\end{array}$ |  |  |  |
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Find the magnitude and the direction of resultant of the forces shown in each of the following figures:



Figure ()

$\mathrm{cm}, 8 \mathrm{~cm}$
Figure ()

The forces $3,6,9 \sqrt{3}$ and 12 kg .wt act on a particle and the measure of the angle between the first and the second is 60 , between the second and the third is 90 and between the third and the fourth is 150 . Find the magnitude and the direction of resultant of these forces.

Three forces of magnitudes $10,20,30$ newton act at a particle. The first acts towards the east and the second makes an angle of measure 30 west of the north and the third makes an angle of measure 60 South of the west. Find the magnitude and the direction of resultant of these forces.

Four forces of magnitudes $10,20,30 \sqrt{3}$ and $40 \mathrm{gm} . \mathrm{wt}$ act on a particle, the first acts in the east direction and the second acts in the direction 60 north of the east and the third acts in the direction 30 north of the west and the fourth acts in the direction making an angle of 60 South of the east. Find the magnitude and direction of resultant of these forces.

ABC is an equilateral triangle, M is the point of intersection of its medians. The forces of magnitudes $15,20,25$ newton act on a particle in the directions of $\overrightarrow{M C}, \overrightarrow{M B}, \overrightarrow{M A}$. Find the magnitude and the direction of the resultant of these forces.

If $\mathrm{F}_{1}=5 i+3 \mathrm{j}, \mathrm{F}_{2}=\mathrm{a} i+6 \mathrm{j}$ and $\mathrm{F}_{3}=14 i+\mathrm{b} \mathrm{j}$ are three coplanar forces meeting at a point and their resultant $\quad \mathrm{R}=(10 \sqrt{2}$, 135) Find the values of $a, b$
(0) In the opposite figure :

If the magnitude of the resultant of the forces equals $3 \sqrt{2}$ Newton, then find the value of F and the measure of the angle between the line of action of the resultant and the first force


In the opposite figure :
If the magnitude of the resultant of the forces equals 20 Kg .wt and acts in the direction of $\overrightarrow{\mathrm{AD}}$ Find the values of F and K .



[^0]:    « 15 newton»

